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NONSTATIONARY ADIABATIC CENTROSYMMETRICAL MOTIONS OF MATTER IN THE GENERAL THEORY OF RELATIVITY

by

S. M. Kolesnikov
K. P. Stanyukovich

(USSR)

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K. P. Stanyukovich

SUMMARY

After deriving the basic equations for the case of nonstationary motions in proper gravitational field and the required concomitant solution of Einstein gravitational field equations (with the included impulse energy conservation equations), the authors offer general solutions for the particular cases of the equations of state, applying them to various problems of cosmology.

• • •

1. – BASIC EQUATIONS.–

In the case of nonstationary motions in proper gravitational fields it is necessary to seek the concomitant solution of Einstein gravitational field equations and of the equations of impulse energy conservation comprised in them [1]

$$R_i^k - \tfrac{1}{2}\delta_i^k = \varkappa T_i^k$$

$$T_{i;k}^{\ k} = \frac{1}{\sqrt{-g}}\frac{\partial(\sqrt{-g}\,T_i^k)}{\partial x^k} - \frac{T^{kl}}{2}\frac{\partial g_{kl}}{\partial x^i} = 0 \qquad (1.1)$$

$$T_i^k = (p+\varepsilon)\,u_i u^k + \delta_i^k p = \frac{W}{v}\,u_i u^k + \delta_i^k p$$

$$\varkappa = \frac{8\pi G}{c^4}, \qquad W = (p+\varepsilon)\,v = E + pv$$

Here R_i^k is the curvature tensor, T_i^k is the impulse energy tensor, $\varkappa$ is the Einstein gravitational constant, W is the heat content per unit

* NESTATSIONARNYYE ADIABATICHESKIYE TSENTRAL'NO-SIMMETRICHNYYE DVIZHENIYA MATERII V OBSHCHEY TEORII OTNOSITEL'NOSTI

of mass, v is the specific volume, u_k — the component four velocities.

If we limit ourselves to the consideration of centrally-symmetrical motions, the time-space metric may be chosen in the following form:

$$\begin{gathered} ds^2 = e^{\nu}c^2dt^2 - e^{\lambda}r^2 - r^2(d\beta^2 + \sin^2\beta\, d\varphi^2) \\ g_{00} = -e^{\nu}, \quad g_{11} = e^{\lambda}, \quad g_{22} = r^2, \quad g_{33} = r^2\sin^2\beta \\ \sqrt{-g} = \exp[{}^1/_2(\nu + \lambda)]\, r^2 \sin\beta \end{gathered} \tag{1.2}$$

In the following we shall study only the radial flows, when $d\beta/dt = 0$, $d\varphi/dt = 0$. In this case, the chronometrically-invariant three-dimensional velocity is given by the expression

$$a^r = \frac{1}{\sqrt{-g_{00}}}\frac{dr}{dt} = e^{-1/2\nu}\frac{dr}{dt}, \qquad a^2 = a_r a^r = g_{11}\left(\frac{dr}{dt}\right)^2 = e^{\lambda-\nu}\left(\frac{dr}{dt}\right)^2$$

with, at the same time,

$$ds^2 = e^{\nu}(c\theta\, dt)^2 = (c\theta\, d\tau)^2, \quad \theta = \sqrt{1 - a^2/c^2}, \quad d\tau = e^{1/2\nu}$$

Here $d\tau$ is the proper time element and the components of 4-velocity will take the form

$$\begin{gathered} u^0 = \frac{1}{\theta}\frac{dt}{d\tau} = \frac{1}{\theta}e^{-1/2\nu}, \qquad u_0 = g_{00}u^0 = \frac{1}{\theta}e^{1/2\nu} \\ u^1 = \frac{dr}{d\tau}\frac{1}{c\theta} = \frac{a}{c\theta}e^{-1/2\lambda}, \quad u_1 = g_{11}u^1 = \frac{a}{c\theta}e^{1/2\lambda}, \quad u_0u^0 + u_1u^1 = -1 \end{gathered} \tag{1.3}$$

The impulse energy conservation equations (1,1) give us the equations of motion and the continuity equation. Since $dW = Td\sigma + dp$, where T is the absolute temperature, σ is the entropy, for the adiabatic processes under consideration it is still necessary to utilize the entropy conservation equation

$$\frac{d(Wu^i)}{ds} + \frac{\partial W}{\partial x^i} = \frac{W}{2}u^k u^l \frac{\partial g_{kl}}{\partial x^i} + T\frac{\partial \sigma}{\partial x^i}, \quad \frac{\partial}{\partial x^k}\left(\frac{\sqrt{-g}\,u^k}{v}\right) = 0, \quad \frac{d\sigma}{ds} = 0 \tag{1.4}$$

The system of equations (1,4) is the complete system of conservation equations characterizing the adiabatic flows. Substituting here the components of 4-velocity (1,3), and taking into account that

$$d(\ln\sqrt{-g}) = \frac{d\lambda + d\nu}{2} + 2\frac{dr}{r} + \operatorname{tg}\beta\, d\beta$$

we shall obtain a system of hydrodynamics equations of radial fllows in the proper gravitational field

$$\frac{1}{(c\theta)^2}\left(A\frac{\partial a}{\partial t}+a\frac{\partial a}{\partial r}\right)-\frac{\omega^2}{c^2}\left(\frac{\partial \ln v}{\partial r}+\frac{aA}{c^2}\frac{\partial \ln v}{\partial t}\right)+\frac{1}{2}\left(\frac{Aa}{c^2}\frac{\partial\lambda}{\partial t}+\frac{\partial\nu}{\partial r}\right)=\frac{\theta^2T}{W}\frac{\partial\sigma}{\partial r}$$

$$-\left(A\frac{\partial \ln v}{\partial t}+a\frac{\partial \ln v}{\partial r}\right)+\frac{1}{\theta^2}\left(\frac{\partial a}{\partial r}+\frac{aA}{\partial r}+\frac{aA}{c^2}\frac{\partial a}{\partial t}\right)+\frac{2a}{r}+\frac{1}{2}\left(A\frac{\partial\lambda}{\partial t}+a\frac{\partial\nu}{\partial r}\right)=0 \qquad (1.5)$$

$$A\frac{\partial\sigma}{\partial t}+a\frac{\partial\sigma}{\partial r}=0, \qquad \frac{\omega^2}{c^2}=-\left(\frac{\partial \ln v}{\partial \ln W}\right)_\sigma, \; A=\exp\left(\frac{\lambda-\nu}{2}\right)$$

Note that the expression for the velocity a^* along the characteristics of these equations has the same form as in the special theory of relativity

$$a^*=A\left(\frac{dr}{dt}\right)^*=\left(\frac{dr}{d\tau_0}\right)^*=\frac{dl}{d\tau}=\frac{a\pm\omega}{1\pm a\omega/c^2}, \quad dl=e^{1/2\lambda}\,dr, \quad d\tau_0=\frac{dt}{A} \qquad (1.6)$$

This, incidently, is quite natural, for the presence of a gravitational field cannot modify the local correlation between the chronometrically-invariant components of the three-dimensional velocities a^*, a, ω, measured at each point r by the hours of the observer situated at the same point. However, as follows from (1,6), the first characteristic at divergence front ($\omega=0$) is found to be rectilinear in variables l, τ, which have a physical sense, while in variables r, t it is curvilinear.

Let us write down the field equations

$$\frac{\partial(re^{-\lambda})}{\partial r}=1+\varkappa r^2T_0^0=1-\frac{\varkappa r^2}{\theta^2}\left(\varepsilon+p\frac{a^2}{c^2}\right)$$

$$A\frac{\partial(re^{-\lambda})}{\partial t}=-\varkappa cAT_0^1=\frac{\varkappa ar^2}{\theta^2}(\varepsilon+p)$$

$$\left(1+r\frac{\partial\nu}{\partial r}\right)e^{-\lambda}=1+\frac{\varkappa r^2}{\theta^2}\left(p+\varepsilon\frac{a^2}{c^2}\right)=1+\varkappa r^2T_1^1 \qquad (1.7)$$

$$\varkappa T_2^2=\varkappa T_3^3=\varkappa p=\frac{e^{-\lambda}}{2}\left[\frac{\partial^2\nu}{\partial r^2}+\frac{1}{2}\left(\frac{\partial\nu}{\partial r}\right)^2+\right.$$

$$\left.+\frac{1}{2}\frac{\partial(\nu-\lambda)}{\partial r}-\frac{\partial\nu}{\partial r}\frac{\partial\lambda}{\partial r}-\frac{A}{c^2}\frac{\partial}{\partial t}\left(A\frac{\partial\lambda}{\partial t}\right)\right]$$

Only two of these equations are independent from equations (1,5). It is quite convenient to write these equations in the form

$$A\frac{\partial\lambda}{\partial t}+a\frac{\partial\lambda}{\partial r}=-a\left(\frac{e^\lambda-1}{r}+\varkappa pre^\lambda\right), \quad A\left(1+\frac{a^2}{c^2}\right)\frac{\partial\lambda}{\partial t}+a\frac{\partial(\lambda+\nu)}{\partial r}=0$$

For the given equation of state $p = p(\varepsilon)$ the equations (1,5) and the first equation (1,8) determine the solution

$$p = p(r,t), \quad \sigma = \sigma(r,t), \quad a = a(r,t), \quad \lambda = \lambda(r,t), \quad \nu = \nu(r,t)$$

1) In the static case $a = 0$ the second equation (1,8) gives

$$\partial\lambda / \partial t = 0,$$

and from (1,5) we have

$$\partial \ln v / dt = 0, \ d\sigma / dt = 0,$$

consequently,

$$\frac{dp}{dr} = -\frac{1}{2}(p+\varepsilon)\frac{d\nu}{dr} \tag{1.9}$$

Subsequently, from equations (1,7) we have

$$\frac{d}{dr}(re^{-\lambda}) = 1 - \varkappa r^2\varepsilon, \qquad e^{-\lambda} r\frac{d\nu}{dr} = \varkappa r^2 p + 1 - e^{-\lambda}$$

Taking advantage of these correlations, we shall eliminate $\nu(r)$ from (1,9)

$$\frac{d}{dr}\left[\frac{r(p+\varepsilon)(1+\varkappa pr^2)}{\{(p+\varepsilon) - 2r\,dp/dr\}}\right] = 1 - \varkappa r^2\varepsilon \tag{1.10}$$

[Note that at $p = \text{const}$ it stems from the equation (1,10) the equation of state of a closed static model of the Einstein Universe

$$\varepsilon + 3p = 0.$$

Hence we conclude that the Einstein model corresponds to a star model with constant negative pressure. From the viewpoint of an outer observer, the closed state of such a star implies the impossibility of crossing the star's boundary by a geodesic line of any signal; it however, does not reflect, in principle, the absence of the boundary. That is why the closed static model may be viewed as an autonomous bounded non-Euclidean formation, immersed in the outer spatial background, and, by way of consequence, the closed state does not imply in any way the uniqueness of this model of Universe.]

Resolving the equation (1,10) we determine $p(r)$, then $\lambda(r)$, $\nu(r)$, which fully resolves the problem about equilibrium.

2) A substantial interest is offered by the study of radial flows of an ideal fluid in the given gravitational field, for example, either the outer or inner Schwarzshild field, when $\lambda = \lambda(r)$, $\nu = \nu(r)$ are the given functions of r. It is easy to see that in this case the equations (1,5) will be reduced, by mere introduction of the independent variable $dr_1 = A\,dr$, to a form analogous to hydrodynamics equations of the special theory of relativity, and they differ from them only by the form of free terms

$$\frac{1}{(c\theta)^2}\left(\frac{\partial a}{\partial t} + a\frac{\partial a}{\partial r_1}\right) - \frac{\omega^2}{c^2}\left(\frac{\partial \ln v}{\partial r_1} + \frac{a}{c^2}\frac{\partial \ln v}{\partial t}\right) + \frac{1}{2}\frac{d\nu}{dr_1} = \frac{T\theta^2}{W}\frac{\partial \sigma}{\partial r_1}$$

$$-\left(\frac{\partial \ln v}{\partial t} + a\frac{\partial \ln v}{\partial r_1}\right) + \frac{1}{\theta^2}\left(\frac{\partial a}{\partial r_1} + \frac{a}{c^2}\frac{\partial a}{\partial t}\right) + \frac{a}{2}\frac{d\nu}{dr_1} + \frac{2a}{r_1}\frac{r_1}{rA} = 0 \tag{1.11}$$

$$\frac{\partial \sigma}{\partial t} + a\frac{\partial \sigma}{\partial r_1} = 0$$

For the outer Schwarzshild field

$$\nu + \lambda = 0, \quad \frac{d\lambda}{dr} + \frac{e^{\lambda} - 1}{r} = 0, \quad e^{-\lambda} = e^{\nu} = 1 - \frac{r_0}{r}, \; r_0 = \frac{2GM_0}{c^2}$$

Hence

$$\frac{d\nu}{dr_1} = \frac{r_0}{r^2(1 - r_0/r^2)^2}$$

Here M_0 is the mass of the central body creating the field.

3) In the general case of adiabatic flows in proper gravitational field the finding of the solution of the aggregate system of equations (1,5) and of the equation (1,8) present substantial difficulties. In the conclusion of the work we shall obtain this solution in the asymptotic case of motions with velocities near the speed of light and the ultrarelativistic equation of state. Here too, we propose a method of consecutive integration of (1,5) and of the first equation (1,8), with the utilization of (1,7).

Let us first of all eliminate from the equations (1,5) the function $\nu = \nu(r, t)$ with the help of the second equation (1,8); we shall obtain

$$\frac{1}{(c\theta)^2}\left(A\frac{\partial a}{\partial t} + a\frac{\partial a}{\partial r}\right) - \frac{\omega^2}{c^2}\left(\frac{\partial \ln v}{\partial r} + \frac{aA}{c^2}\frac{\partial \ln v}{\partial t}\right) = \frac{1}{2a}\left(A\frac{\partial \lambda}{\partial t} + a\frac{\partial \lambda}{\partial r}\right) + \frac{\theta^2 T}{W}\frac{\partial \sigma}{\partial r}$$

$$-\left(A\frac{\partial \ln v}{\partial t} + a\frac{\partial \ln v}{\partial r}\right) + \frac{1}{\theta^2}\left(\frac{\partial a}{\partial r} + \frac{aA}{c^2}\frac{\partial a}{\partial t}\right) +$$

$$+\frac{2a}{r} = \frac{a}{2}\left(\frac{\partial \lambda}{\partial r} + \frac{aA}{c^2}\frac{\partial \lambda}{\partial t}\right) \tag{1.12}$$

In the equations (1,12) we shall pass to independent variables r,λ; to that effect we shall find with the aid of the first equation (1,7) and of the first equation (1,8) the Jacobian transformations $\partial(t;r)/\partial(\lambda;r)$, and also $\partial t/\partial r$; we have

$$a\frac{\partial(t;r)}{\partial(\lambda;r)} = a\frac{\partial t}{\partial\lambda} = -\frac{AA_1 r}{A_4}, \qquad a\frac{\partial t}{\partial r} = A\left(1-\frac{A_2}{A_4}\right) \tag{1.13}$$

Here

$$A_1 = e^{-\lambda}, \quad A_2 = 1 + \varkappa p r^2 - e^{-\lambda}, \quad A_3 = \varkappa r^2 \varepsilon + e^{-\lambda} - 1, \quad A_4 = \varkappa(p+\varepsilon)(r/\theta)^2$$

The correlations (1,13) bring the equations (1,12) to the symmetric form

$$\frac{1}{2(\theta c)^2}\left(A_1\frac{\partial a^2}{\partial \ln r} - A_2\frac{\partial a^2}{\partial\lambda}\right) - \frac{\omega^2}{c^2}\left(A_1\frac{\partial \ln v}{\partial \ln r} + A_3\frac{\partial \ln v}{\partial\lambda}\right) + \frac{1}{2}A_2 = \frac{T\theta^2 A_4}{W}\frac{\partial \sigma}{\partial\lambda}$$

$$\left(A_1\frac{\partial \ln v}{\partial \ln r} - A_2\frac{\partial \ln v}{\partial\lambda}\right) - \frac{1}{\theta^2}\left(A_1\frac{\partial \ln v}{\partial \ln r} + A_3\frac{\partial \ln a}{\partial\lambda}\right) - $$

$$-2A_1 + \frac{A_3}{2} = 0, \quad A_2\frac{\partial\sigma}{\partial\lambda} - A_1\frac{\partial\sigma}{\partial \ln r} = 0 \tag{1.14}$$

Let us recall that for the selected equation of state, for example, at $pv^k = \sigma, \omega, \varepsilon$ and p, the functions ϑ, σ and the equations (1,4) contain the three unknown functions

$$v = v(\lambda, r), \qquad a = a(\lambda, r), \qquad \sigma = \sigma(\lambda, r)$$

The integration of these equations may be performed by the method of characteristics.

After that we determine $\nu = \nu(\lambda, r)$ from the first equation (1,8) which, in variables λ, r takes the form

$$A_1\frac{\partial\nu}{\partial r} + (A_4 - A_2)\frac{\partial\nu}{\partial r} = \frac{a^2}{c^2}A_4 + A_2 \tag{1.15}$$

Finally, with the aid of (1,13) we find $t = t(\lambda, r)$, which gives us the complete solution of the problem.

Thus, the consecutive integration of (1,14), (1,13) and (1,15) allows to construct the solutions of isotropic motions in the reading system linked with the outlined symmetry center and not in the concomitant calculation system where $a = 0$. The three-dimensional velocity a, measured in such reading systems, has a specific physical sense when the statement of boundary problems is made in the given outer gravitational field. as well as study of motion in proper graviattional field.

Moreover, the equations (1,5) pass in the limit case of gravitational field absence (Galilee metric) to hydrodynamics equations of the special theory of relativity, whereas in the concomitant reading system such a transition is inconsistent. inasmuch as the latter is precisely determined from the condition of equality to zero of impulse energy flux. In case of isentropic flows, the system (1,14) is reduced to two equations.

2. — GENERAL SOLUTIONS OF PARTICULAR CASES OF EQUATIONS OF STATE

a) Dust-like Matter *. - In cosmological problems it is usually admitted that the pressure vanishes little by comparison with the mean density of matter in the Universe, that is, $p \ll \varepsilon = \rho c^2$. If we postulate $p = 0$, $\sigma =$ const we have $\omega = 0$, $d \ln \nu = - d \ln \varepsilon$, and the equation (1,14) will be considerably simplified

$$\frac{1}{(\theta c)^2}\left(\alpha \frac{\partial a^2}{\partial \lambda} - A_1 \frac{\partial a^2}{\partial \ln r}\right) = \alpha, \qquad \alpha = 1 - A_1$$

$$A_1 \frac{\partial \ln \varepsilon}{\partial \ln r} - \alpha \frac{\partial \ln \varepsilon}{\partial \lambda} + \frac{1}{\theta^2}\left(A_1 \frac{\partial \ln a}{\partial \ln r} + A_3 \frac{\partial \ln a}{\partial \lambda}\right) + 2A_1 - \frac{A_3}{2} = 0 \tag{2.1}$$

The first equation of this system is easily integrated

$$\theta^2 = 1 - a^2/c^2 = A_1 \Phi_1(\alpha r) \tag{2.2}$$

Subsequently, from the second equation (2,1) we find

$$\varepsilon = (\Phi_2(\alpha r) - B_2^{-1}) B_1, \qquad B_1 = \exp\left(\int B_3 \, d\lambda\right), \qquad B_2 = \int B_4 B_1 \, d\lambda,$$
$$B_3 = \alpha\left(\frac{1}{2\Phi_1} - \frac{3A_1 + 1}{2}\right) \tag{2.3}$$
$$B_4 = -\frac{\varkappa \alpha^3 r^2}{(1 - A_1\Phi_1)}\left\{\frac{r}{\Phi_1}\frac{d\Phi_1}{d(\alpha r)} + \frac{\Phi_1}{\alpha^2}(1 - A_1\Phi_1) - 1\right\}$$

Utilizing (2,2) and (2,3), we obtain for the function $\nu(r, \lambda)$ from the equation (1,15) at $p = 0$

$$A_1 \frac{\partial \nu}{\partial r} + \frac{\partial \nu}{\partial \lambda}\left(\frac{\varkappa r \varepsilon}{\theta^2} - \frac{\alpha}{r}\right) = \frac{a^2 \varkappa r \varepsilon}{(c\theta)^2} + \frac{\alpha}{r} \tag{2.4}$$

This equation is also resolved in quadratures. The characteristics' equation

$$\frac{\partial \lambda}{\partial r} = e^{\lambda}\left(\frac{\varkappa r B_1(\Phi_2 - B_2^{-1})}{\Phi_1} - \frac{\alpha}{r}\right)$$

---

* The problem of gas flow at $p = 0$ in the concomitant reading system was first resolved by Tolman in 1934 [1] . However in this solution the velocities for the intrinsic Schwartzshild problem were not computed.

gives $\chi_1(\lambda, r)$.- The second integral determines the solution along the characteristics

$$\nu(\lambda, r) = \frac{\varkappa}{A_1}\int \frac{rB_1}{\Phi_1}(1 - A_1\Phi_1)(\Phi_2 - B_2^{-1})\,dr + \alpha \ln r + \chi(\chi_1(\lambda, r)) \tag{2.5}$$

Finally, the first equation (1,13) assigns the last quadrature

$$t = -\frac{1}{\varkappa rc}\int \frac{\Phi_1 \exp\{-{}^1/_2[\lambda + \nu(\lambda, r)]\}\,d\lambda}{\sqrt{1 - A_1\Phi_1(\Phi_2 - B_2^{-1})B_1}} + \psi(r) \tag{2.6}$$

Therefore, we obtained the general solution of the problem of motion of the dust-like matter, given by the integrals (2,2)— (2,6) and depending upon four arbitrary functions $\Phi_1, \Phi_2, \psi, \chi$.

When resolving concrete boundary problems, it is necessary to assign the preliminary distribution of velocities $\theta_0 = \theta_0(\lambda, r)$ and determine Φ_1; it is then easy to find Φ_2, ψ, χ.

b) Ultrarelativistic Approximation.- Let us consider the case of adiabatic motions with velocities near the speed of light. We postulate $a/c = 1 - 2\Delta$, where $\Delta \ll 1$. Neglecting the higher orders of smallness of Δ, we shall write the systems (1,5) and (1,8) in the form

$$A\frac{\partial \ln(Wv)}{\partial t'} + \frac{\partial \ln(Wv)}{\partial r} = \frac{2}{r}, \qquad A\frac{\partial \lambda}{\partial t'} + \frac{\partial \lambda}{\partial r} = -\left[\frac{e^{\lambda} - 1}{r} + \varkappa p r e^{\lambda}\right]$$
$$A\frac{\partial \Pi}{\partial t'} + \frac{\partial \Pi}{\partial t'} = 0, \qquad A\frac{\partial \sigma}{\partial t'} + \frac{\partial \sigma}{\partial r} = 0,\ t' = ct,\ \Pi = \ln\Delta + \lambda - 2\ln W \tag{2.7}$$

Note that

$$d\ln(Wv) = \frac{\partial \ln(Wv)}{\partial p}dp + \frac{\partial \ln(Wv)}{\partial \sigma}d\sigma$$

The first equation of this system, taking into account the fourth, gives

$$A\frac{\partial p}{\partial t'} + \frac{\partial p}{\partial r} = \frac{2}{r}\left(\frac{\partial p}{\partial \ln(Wv)}\right)_{\sigma} \tag{2.8}$$

We shall transform the third equation (2,7) analogously

$$A\frac{\partial(\ln\Delta+\lambda)}{\partial t'}+\frac{\partial(\ln\Delta+\lambda)}{\partial r}-2\left(\frac{\partial\ln W}{\partial p}\right)_\sigma\left(A\frac{\partial p}{\partial t'}+\frac{\partial p}{\partial r}\right)=0 \tag{2.9}$$

For the relativistic equation of state

$$p=(k-1)\varepsilon,\qquad Wv=v(\varepsilon v+pv)=\frac{kpv^2}{k-1}.$$

Hence, taking into account that $pv^k=\sigma$, we find

$$\left(\frac{\partial\ln W}{\partial p}\right)_\sigma=\frac{k-1}{kp},\qquad \left(\frac{\partial\ln(Wv)}{\partial p}\right)_\sigma=\frac{k-2}{kp}$$

Substituting these values into the equations (2,7) and passing to independent variables p, r, we shall obtain the system

$$\begin{gathered}\frac{\partial\lambda}{\partial r}+\frac{e^\lambda-1}{r}+\varkappa rpe^\lambda-b\frac{\partial\lambda}{\partial p}=0\\ \frac{\partial(\ln\Delta+\lambda)}{\partial r}-b\frac{\partial(\ln\Delta+\lambda)}{\partial p}+\frac{4(k-1)}{(2-k)r}=0\\ \frac{\partial\sigma}{\partial r}-b\frac{\partial\sigma}{\partial p}=0,\quad A-\frac{\partial t'}{\partial r}+b\frac{\partial t'}{\partial p}=0,\quad b=\frac{2kp}{(2-k)p}\end{gathered} \tag{2.10}$$

The first equation is integrated at once

$$1-e^{-\lambda}=\frac{1}{r}F_1(\gamma)-c_1,\quad \gamma=pr^{2k/2-k},\quad c_1=\frac{\varkappa pr^2(2-k)}{6-5k} \tag{2.11}$$

After that it is easy to write the solution of the second equation (2,10)

$$\Delta=e^{-\lambda}F_2(\gamma)\,p^{2(k-1)/k}=\left(1-\frac{1}{r}F_1+c_1\right)F_2p^{2(k-1)/k} \tag{2.12}$$

The third equation (2,10) gives

$$\sigma=F_3(\gamma) \tag{2.13}$$

The fourth equation (2,10) may be resolved after the determination of $\nu=\nu(p,r)$, since $A=\exp[{}^1/_2(\lambda-\nu)]$. For the determination of ν we shall utilize the second equation (1,8), written in the variables p, r; at the same time, in our approximation

$$2A\frac{\partial\lambda}{\partial p}-\frac{\partial t'}{\partial r}\frac{\partial(\lambda+\nu)}{\partial p}+\frac{\partial t'}{\partial p}\frac{\partial(\lambda+\nu)}{\partial r}=0 \tag{2.14}$$

We shall substitute the derivative $\partial t' / \partial r$ from the fourth equation (2,10), and we shall determine $\partial t' / \partial p$ from the second equation (1,7)

$$2 \frac{\partial \lambda}{\partial p} - \frac{\partial (\lambda + \nu)}{\partial p} = e^{\lambda} \frac{\partial \lambda}{\partial p} \frac{4(k-1)\Delta}{\varkappa k p^2} \left[\frac{\partial (\lambda + \nu)}{\partial r} - b \frac{\partial (\lambda + \nu)}{\partial p} \right]$$

This equation assigns $\nu = \nu(p, r)$, following which we shall write the last quadrature for

$$ct = F_4(\gamma) + \int \exp [{}^1/_2 \{\lambda (p, r) - \nu (p, r)\}] \, dr \qquad (2.15)$$

The constructed solution depends on five arbitrary functions and resolves the probel set up.

[It may be said that the basic equations (1,5) and (1,7), which allow the inclusion into consideration the electromagnetic fields too, fully describe the cemtrosymmetrical flows in proper gravitation field and may be utilized in cosmology of isotropic space. The general theory of relativity in this context is simply the gas dynamics of the Riemannian space.

It should be noted that the problem set up here of investigation of exact equations, practical for the dewcription of the relativistic motion of the medium in proper gravitational field, may be resolved by utilizing the variational methods of the continuum and the field equations [2]].

*** THE END ***

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213	KATZOFF
235	SEATON
185	WEATHERWAX [2]

J P L

SNYDER
NEWBURN
WYCKOFF

UCLA

COLEMAN

UC BERKELEY

WILCOX

U. IOWA

VAN ALLEN

WPAFB

TDBX-T